

# STAINLESS STEEL – PLASTICITY AND CONSTITUTIVE MODELLING

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## Abstract

*This paper presents a constitutive model proposed for stainless steel. The model is a two surface model utilizing the concept of fuzzy sets. An experimental investigation has been performed on two different stainless steel grades as a reference to the model. The tests were performed with a procedure containing load reversal. Each specimen was initially loaded in one direction of the principal stress plane followed by unloading and subsequent loading in a new direction. The model is relatively simple but still depicts the effects of observed phenomena such as the Bauschinger effect. Hence the qualitative response to subsequent loadings can be described with the model. The proposed model has been implemented into the finite element package ABAQUS. Comparisons between test results and the response predicted utilizing the model are presented in this paper.*

## 1 INTRODUCTION

Finite element analyses are utilised as a standard design tool in many disciplines where structural design is performed, e.g. vehicle crash simulations and advanced structural steelwork. Although the areas of application might be far apart, the simulations often have a lowest common denominator; the description of the mechanical material behaviour is not taken into account or even reflected upon, but the default option of the FE-package is used. Immense amounts of research have been performed, but simple constitutive models for the material response are often still used. Whilst it is true that research has resulted in models that can depict various responses, the test data required for application are missing.

Work on the plastic behaviour of stainless steel commenced at the Division of Steel Structures, Luleå University of Technology, as part of an ECSC supported project in 1996. It was a continuation of similar work on structural steel that had been ongoing at the division since the beginning of the 1990s. The first part of the work on stainless steel comprised an extensive test programme in which the austenitic grades 1.4301 (AISI 304), 1.4436 (AISI 316) and the duplex grade 1.4462 (2205), all in annealed condition, were considered. Test results and application of a constitutive model developed for structural steel, see Granlund [1], were presented at a stainless steel experts seminar in 1998 [2]. It was however concluded that in addition to the features of the constitutive model applied, the option of initial anisotropy would improve the agreement between test results and model predictions, especially for the duplex grade. The anisotropy is a feature also, to a varying degree, evident for work hardened stainless steels.

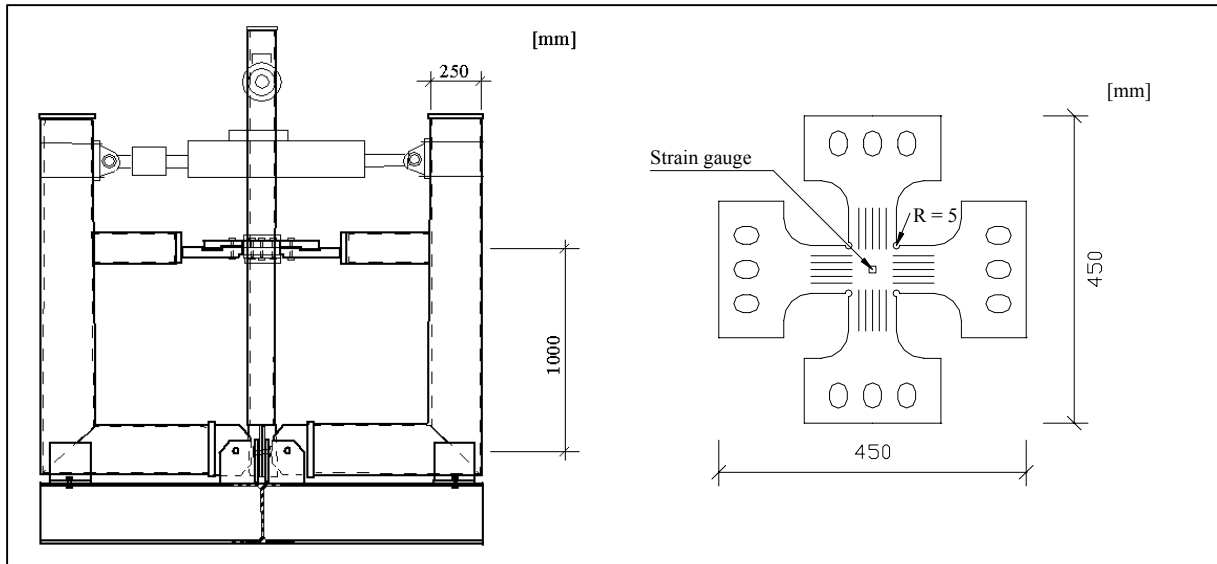
This paper addresses and briefly describes progress in the area of non-monotonic biaxial testing and constitutive modelling following work carried out at Luleå University of Technology between 1998 and 2003. The work comprises development of the constitutive model applied in [2] and [3], enabling modelling of grades exhibiting initial anisotropy and experimental work on work hardened austenitic stainless steel, grade 1.4318.

## 2 EXPERIMENTAL WORK

The concept for biaxial testing developed at the Division of Steel Structures, Luleå University of Technology, and utilised for the work presented in [1], [2], [3] and [4] was used also in the experimental investigation resulting in the test results presented herein. In this biaxial testing method flat specimens were tested, i.e. the material was tested in its most commonly delivered condition.

The testing rig has four hinged arms and two actuators in perpendicular directions, see Figure 1. Compression tests are enabled by the use of support plates clamped around the specimen and guided by the grips of the test rig. The bolts holding the support plates are equipped with strain gauges allowing tests to be performed under identical conditions.

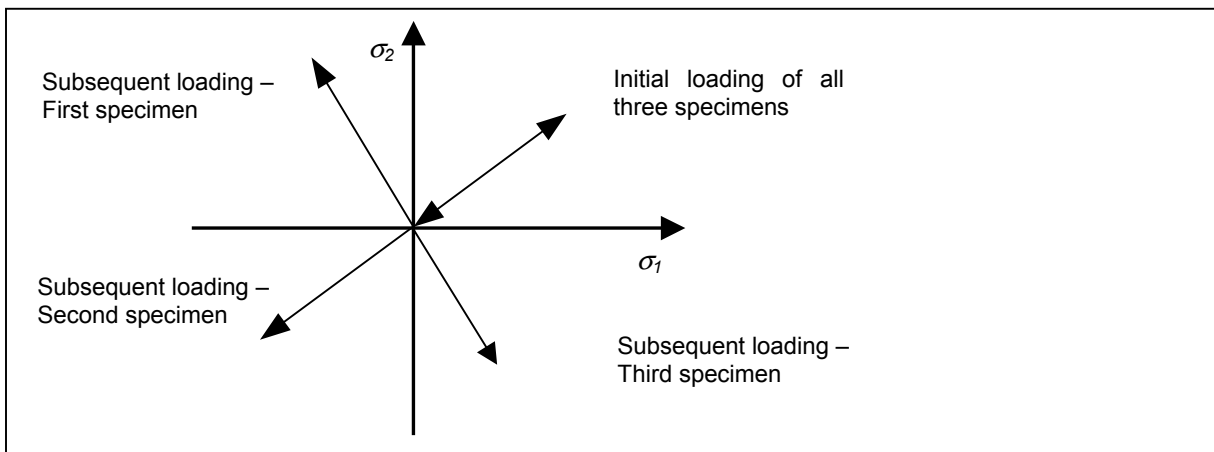
The cross shaped specimen was designed with emphasis on obtaining a homogenous strain distribution in the centre area of the specimen where the strains are measured, limiting the movements of the grips of the testing rig and minimizing the force escaping into the specimen arms perpendicular to loading. These requirements were met by a design according to Figure 1.



**Figure 1** Test rig and cross-shaped specimen for biaxial testing.

Friction losses due to the support plates were considered in the test evaluation. The agreement between uniaxial tests and corresponding tests using the biaxial testing concept is good. See e.g. [4] for a comparison. Hence the validity of the concept for testing is verified.

The materials considered in this paper are the austenitic stainless grade 1.4318 (AISI 301LN) at two different levels of cold work, C700 and C850, and the ferritic-austenitic grade 1.4462. A total of 36 tests were performed on 3 mm thick 1.4318 and 18 tests on 4 mm thick 1.4462. The tests were divided into sets of three. All specimens in each set were initially loaded in the same direction of the principal stress plane  $\sigma_1 - \sigma_2$  and subsequently loaded at the sum of the initial direction plus  $90^\circ$ ,  $180^\circ$  and  $270^\circ$  respectively, see Figure 2. Initial and subsequent loading directions applied to grade 1.4318 are shown in Table 1, for grade 1.4462 see [3] and [4].



**Figure 2** Principle of initial and subsequent loadings within a set of tests.

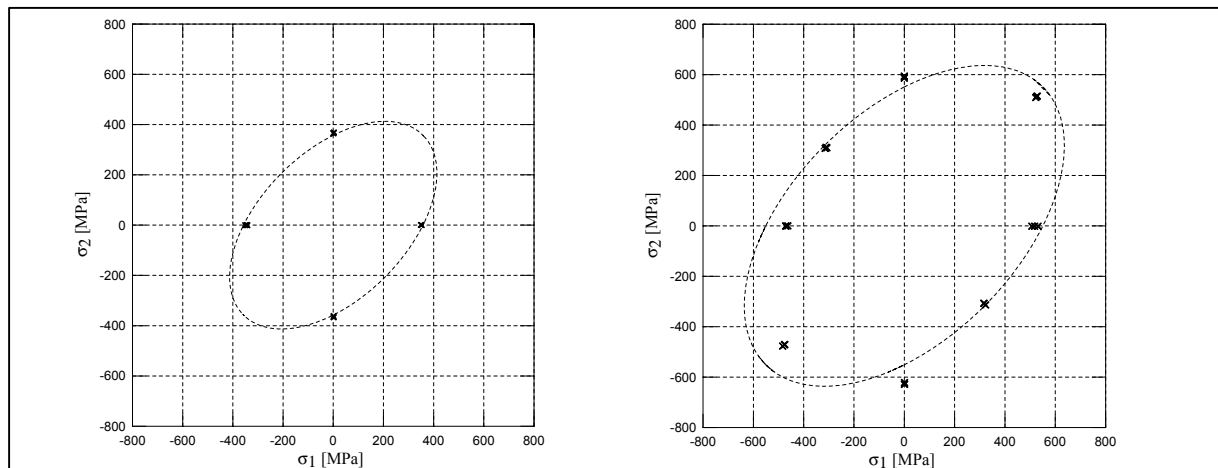
**Table 1** Test programme for the biaxial tests performed on grade 1.4318.

Grade	Initial loading direction	Subsequent loading directions		
		Test 1	Test 2	Test 3
<b>1.4318 C700</b>	0°	90°	180°	270°
	90°	180°	270°	0°
	180°	270°	0°	90°
	270°	0°	90°	180°
<b>1.4318 C850</b>	0°	90°	180°	270°
	45°	135°	225°	315°
	90°	180°	270°	0°
	135°	225°	315°	45°
	180°	270°	0°	90°
	225°	315°	45°	135°
	270°	0°	90°	180°
	315°	45°	135°	225°

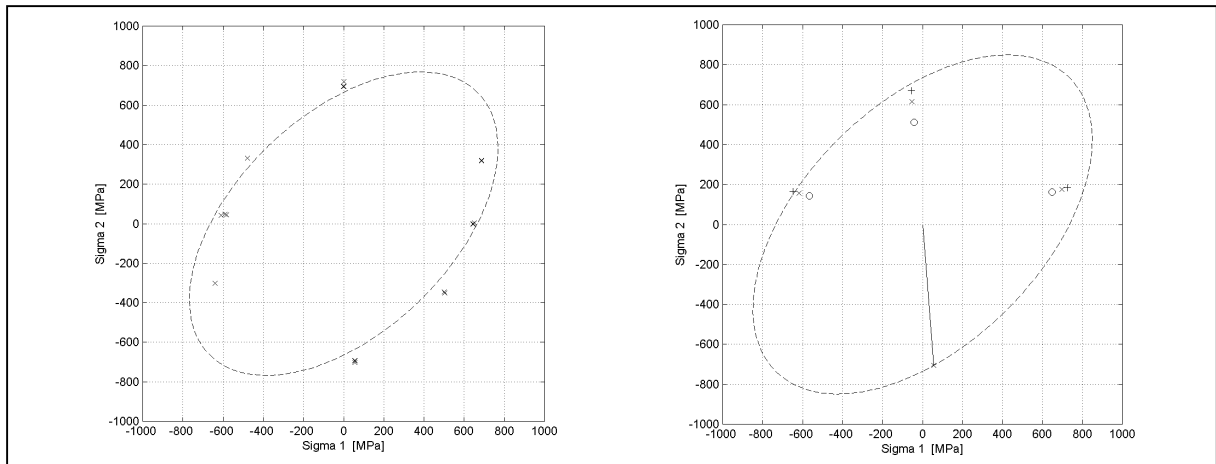
Loading according to Table 1 allows for evaluation of the initial and subsequent yield criteria as well as parameters necessary to apply the constitutive model presented in [4].

The behaviour of 1.4318 C700 was reasonably isotropic, as can be seen by the initial yield criteria, defined as  $R_{P0.2}$ , shown on the principal stress plane in Figure 3. When the material is cold worked, 1.4318 C850, a clear anisotropy is developed. This can be seen in Figure 3 where the agreement between test results and the isotropic yield surface is not as good. For the duplex grade, the initially anisotropic behaviour is evident, as can be seen in Figure 4.

The response to subsequent loadings is characterized by an evident Bauschinger effect as well as strain hardening perpendicular to the initial loading, i.e.the cross effect, see [4].



**Figure 3** Initial yield criteria defined as 0.2 % proof stress,  $R_{P0.2}$ , for 1.4318 C700 and C850. A von Mises loci corresponding to the mean proof stress of the tests is given as a reference.



**Figure 4** Initial yield criteria defined as 0.2 % proof stress,  $R_{P0.2}$ , for 1.4462, to the right. A von Mises loci corresponding to the mean proof stress of the tests is given as a reference. To the left a test set including both initial loading in  $275^\circ$  and subsequent loadings for 1.4462 is shown.

### 3 THE CONSTITUTIVE MODEL

The constitutive model proposed for stainless steel is a development of the model proposed for structural steel by Granlund [1]. The features, compared with [1], those that have been added are, initial anisotropy and a more general potential surface governing the distortion induced by plastic strains. The mathematical description of the model is rather extensive and cannot be given here. See Olsson [4].

The concept of two surfaces is utilised as a foundation for the model, one elastic limit surface bounding the region in the stress space assumed completely elastic and one used as a memory surface. In many ways the concepts of the proposed model are similar to earlier proposed two-surface models. There are however some features that are specific. The transition from elastic to plastic state in loadings following the first is described using the concept of fuzzy sets, introduced into plasticity by Klisinski 1988 [5]. Also the meaning of the elastic limit surface differs from classical theory of plasticity. The elastic limit surface bounds the region assumed to be completely elastic but does not correspond to a yield surface in a classical concept. In a non-monotonic loading the stress point is allowed to move outside the elastic limit surface.

From the experimental study there are some features in the mechanical response of the materials that are important to depict in order to reflect the phenomenological observations made: the pronounced Bauschinger effect, i.e. a reduction of strength in a direction opposite to initial loading, the cross effect, i.e. the increased strength in a direction transverse to the initial loading direction, the gradual transition from elastic to plastic state at loadings following the initial loading and initial anisotropy.

The proposed constitutive model has in [4] been shown capable of reflecting the phenomena mentioned above and fulfils the requirement of relative simplicity. By simplicity, it is here meant that the number of parameters should be kept at a minimum. Generally, it is possible to formulate very accurate models using a large number of parameters, but as model parameters usually are experimentally determined, the usefulness of a model is coupled to the work needed to determine its parameters.

#### 3.1 A general description of the proposed model

The memory surface is an isotropically expanding distorted von Mises, or von Mises-Hill, surface that is used to keep track of the largest effective stress the material has been subjected to and the plastic modulus associated with that stress. It also governs the direction of plastic flow at loadings where plastic strains occur.

The elastic limit surface bounds the domain where a totally elastic response is assumed and is a combination of two surfaces. In the region of the stress point, a von Mises - Hill surface that hardens according to a mixed hardening rule is used, and in the region opposite the stress point, a distorted surface

is used. In addition, initial anisotropy is reflected also in the elastic limit surface. The elastic limit surface is not allowed to intersect the memory surface.

At reloading, two separate cases can be distinguished. Reloading in the region of initial loading, the I-region, or reloading in the region opposite the initial loading, the II-region. When reloading in the I-region the elastic limit surface remains in the same position while the stress point moves towards the memory surface. As the memory surface is reached the elastic limit surface translates towards the stress point such that the surfaces are tangential at contact. When reloading in the II-region the stress point moves outside the elastic limit surface, which remains, but gradually loses its distortion. When the memory surface is reached by the stress point the elastic limit surface will be undistorted and start to translate towards the stress point in the same way as in the I-region. As it translates towards the memory surface plastic strains are generated and the elastic limit surface is distorted in the region opposite the stress point. When the memory surface is reached the mixed hardening rule is applied again.

When the stress point is located between the elastic limit surface and the memory surface, i.e. in the yielding region, the fuzzy surface, here renamed as transition surface, is utilised for the gradual transition from elastic to plastic state. For each stress point  $\sigma_{ij}$  in this yielding region there exists a real value  $\gamma$  on the interval (0,1) such that if  $\gamma=1$ , the stress point is on the elastic limit surface and the plastic modulus has a large value, theoretically equal to infinity. If  $\gamma=0$ , the stress point is on the memory surface and the plastic modulus corresponds to the size of the memory surface. This means that as the stress point moves through the yielding region the value of  $\gamma$  varies from 1 towards 0 and the plastic modulus varies according to some function  $f(\gamma)$  from an initial large value towards the plastic modulus corresponding to the size of the memory surface.

Initial anisotropy is defined by the distortional tensor  $b_{ij}^0$ . The yield stress in the different directions are expressed as a relation according to the proposal by Hill [4].

$$R_{ij} = \frac{\sigma_{ij}}{\sigma_0}$$

where  $\sigma_0$  is the chosen reference yield stress. For plane stress the initial yield criterion can be simplified to

$$\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 + \frac{1}{6} \left[ b_x^0(2\sigma_x - \sigma_y) + b_y^0(2\sigma_y - \sigma_x) + 6a_{xy}^0\tau_{xy} \right]^2} - \sigma_0 = 0$$

from which the vector  $\mathbf{b}^0$  governing the anisotropy can be obtained as

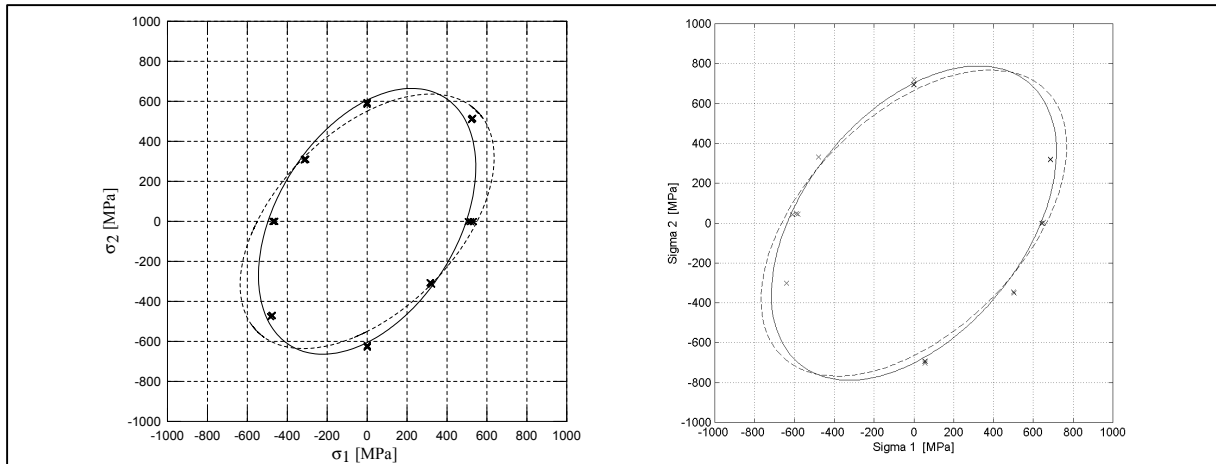
$$\mathbf{b}^0 = \begin{bmatrix} b_x^0 \\ b_y^0 \\ a_{xy}^0 \end{bmatrix} = \begin{bmatrix} (2A - B)/3 \\ (A - 2B)/3 \\ C \end{bmatrix}$$

where

$$A = \sqrt{\frac{6}{R_{11}^2} - 6}; \quad B = \sqrt{\frac{6}{R_{22}^2} - 6}; \quad C = \sqrt{\frac{3}{2R_{12}^2} - \frac{1}{2}}$$

The effects of different yield strengths in the rolling direction and transverse the rolling direction can now be taken into account.

In Figure 5, stress points corresponding to 0.2 % plastic strain at initial loading are compared with an isotropic as well as an anisotropic yield criterion. As can be seen in Figure 5, considering anisotropy as proposed, an improvement compared with isotropic yield criteria is achieved.



**Figure 5** Experimental results,  $R_{P0.2}$ , for 1.4318 C850 and 1.4462. Initial yield criteria, anisotropic (solid) and isotropic (dashed).

Compared to a single surface model with isotropic hardening, the proposed constitutive model requires only four additional parameters and in the case of initial anisotropy five additional parameters. The parameters needed are listed below and the first two are needed for any model.

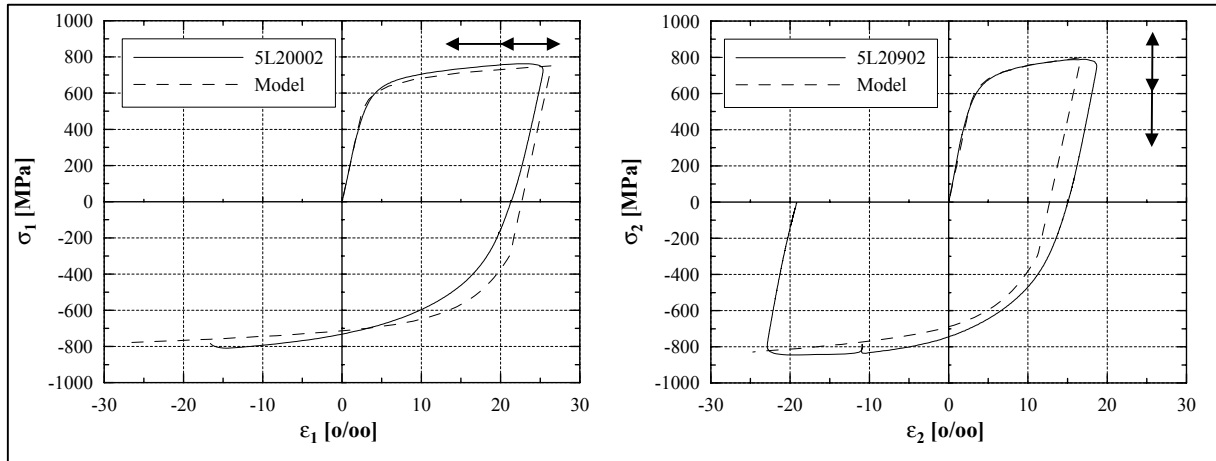
- A description of the uniaxial stress – strain curve in a reference direction.
- The initial elastic limit  $\sigma_0$ .
- Stress ratios  $R_{ij} = \sigma_{ij} / \sigma_0$ .

These three parameters can be evaluated from standard uniaxial tensile coupon tests, whereas the following parameters, at least initially, need to be evaluated from biaxial tests.

- The parameter  $M$  governing the relationship between isotropic and kinematic hardening for the elastic limit surface.
- The parameter  $\psi$  governing the distortion of the elastic limit surface.
- The angle  $\theta_i$  governing the shape of the distortion surface.
- The function modifying the generalized plastic modulus with respect to  $\gamma$  in subsequent loadings.

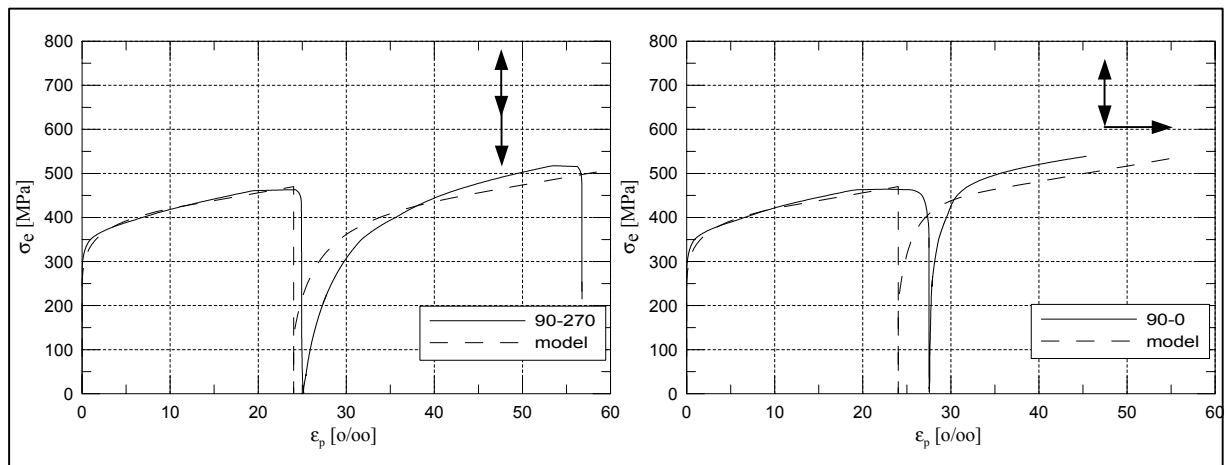
### 3.2 Comparison between tests and proposed model

In this section, the predicted response using the proposed constitutive model is compared with test results. The importance of depicting the anisotropy in the model is shown in Figure 6 where tensile loadings parallel (left graph) and transverse (right graph) to the rolling direction are shown. In both cases, the subsequent loading is opposite to the initial, i.e. compression in and transverse to the rolling direction respectively. Comparing the two loadings, it can be seen that an assumption of isotropic hardening with the transverse direction as reference would result in an over estimation of the response parallel to the rolling direction. Thus it follows that since the behaviour for an anisotropic material is different depending on loading direction it would be difficult to depict the behaviour without including initial anisotropy.



**Figure 6** Comparison between tests and model predictions for grade 1.4462. Left: Initial loading along the rolling direction followed by compression in the same. Right: Initial loading transverse to the rolling direction followed by compression transverse the rolling direction.

Considering tests on grade 1.4318 C700, for which the effective stress – effective plastic strain response is compared with the corresponding model predictions in Figure 7. The graphs show that the proposed model can be utilised to describe the qualitative features of the response to the subsequent loadings.



**Figure 7** Comparison between tests on the grade 1.4318 C700 initially loaded in  $90^\circ$ , i.e. transverse the rolling direction, and the proposed model.

## 4 DISCUSSION AND CONCLUSIONS

Considering the results from the biaxial testing, it is clear that the Bauschinger effect and the cross effect are the most pronounced phenomena affecting the subsequent yield criteria. Especially the Bauschinger effect is introduced rapidly as plastic strains occur. These features are bound to affect the response to subsequent loadings and the transition from elastic to plastic state was consequently observed as both gradual and strongly direction dependent. It is thus evident that anisotropy resulting from initial loadings is important when studying loadings where stress reversals can occur. For the duplex grade, the anisotropy of the initial yield criterion is pronounced and the agreement with the isotropic criterion is consequently not so good. For the austenitic grade, the initial anisotropy as expected depends on the level of work hardening, i.e. a higher level of work hardening results in a more pronounced initial anisotropy.

The constitutive model proposed for application to stainless steel has the possibilities to qualitatively depict the experimentally observed phenomena. Phenomena that traditional models fail to describe, e.g. the Bauschinger effect as well as the cross effect and their consequences on the mechanical response to

subsequent loadings. Comparisons between simulations and test results, in general, show good agreement with respect to initial and subsequent yield criteria as well as stress-strain response. The qualitative improvement compared to simulations obtained using traditional constitutive models is significant. Neglecting initial anisotropy in the model formulation may result in quantitative errors on the unsafe side. This is mainly due to the fact that the initial elastic limit is referred to the transverse direction of the sheet, which as the strength is lower in the rolling direction, would have resulted in an overestimation of the strength in the rolling direction. Furthermore, the formulation of the constitutive model is such that it still is relatively simple, especially when reloading occurs. It would be possible to obtain good results for pure reversed loadings that often are of interest in cyclic plasticity using a traditional two surface model without distortion. However, if model parameters are optimised for such a loading, simulations of other more general loadings, will most likely fail to describe the response, even qualitatively.

A possible improvement of the constitutive model proposed is to enable modelling of cyclic loadings, a feature currently not included in the formulation. Applications of interest for such a model are, for example, structures subjected to cyclic loadings of a magnitude resulting in plastic strains.

The following conclusions are drawn from the work presented:

- The concept for biaxial testing of flat cruciform specimen is applicable to annealed as well as work hardened stainless steel grades and tests are possible in the full principal stress plane within a strain range of approximately  $\pm 3-4$  %.
- The initial yield criteria for the duplex grade and the austenitic grade with a higher level of work hardening show a pronounced anisotropy. The subsequent yield criteria show a pronounced Bauschinger effect even for initial loadings resulting in small plastic strains for all grades. The transition from elastic to plastic state in subsequent loadings is gradual and strongly direction dependent.
- The constitutive model presented can depict the experimentally observed phenomena. Compared to commonly used constitutive models, large improvements both qualitatively and quantitatively in predicting the response to subsequent loading have been shown.

## 5 REFERENCES

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